Fill Ups, True/ False of Differentiation

Fill in the Blanks

Q. 1. If $y = f\left(\frac{2x-1}{x^2+1}\right)$ **and f '(x) = sin x^2 , then** $\frac{dy}{dx} =$ **..................**

Ans. $\frac{2+2x-2x^2}{(x^2+1)^2} \sin\left(\frac{2x-1}{x^2+1}\right)^2$

Solution.

$$\frac{2+2x-2x^2}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$$

$$\text{Given: } y = f\left(\frac{2x-1}{x^2+1}\right); f'(x) = \sin x^2$$

$$\therefore \quad \frac{dy}{dx} = f'\left(\frac{2x-1}{x^2+1}\right).\frac{d}{dx}\left(\frac{2x-1}{x^2+1}\right)$$

$$= \sin\left(\frac{2x-1}{x^2+1}\right)^2.\frac{2(x^2+1)-2x(2x-1)}{(x^2+1)^2}$$

$$= \frac{2+2x-2x^2}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$$

Q. 2. If f_r (x), g (x), h_r (x) r , r = 1, 2, 3 are polynomials in x such that fr(a) = g_r (a) = h_r(a), r = 1, 2, 3

$$\text{and } F(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix} \text{ then } F'(x) \text{ at } x = a \text{ is}$$

Ans. 0

Solution.

$$\text{Given that } F(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix} \qquad \ldots(1)$$

Where $f_r(x)$, $g_r(x)$, $h_r(x)$, r = 1, 2, 3, are polynominals in x and hence differentiable and

$f_r(a) = gr(a) = hr(a)$, r = 1, 2, 3 … (2)

Differentiating eq. (1) with respect to x, we get

$$F'(x) = \begin{vmatrix} f_1'(x) & f_2'(x) & f_3'(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$$

$$+\begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1'(x) & g_2'(x) & g_3'(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1'(x) & h_2'(x) & h_3'(x) \end{vmatrix}$$

$$\therefore \quad F'(a) = \begin{vmatrix} f_1'(a) & f_2'(a) & f_3'(a) \\ g_1(a) & g_2(a) & g_3(a) \\ h_1(a) & h_2(a) & h_3(a) \end{vmatrix}$$

$$+\begin{vmatrix} f_1(a) & f_2(a) & f_3(a) \\ g_1'(a) & g_2'(a) & g_3'(a) \\ h_1(a) & h_2(a) & h_3(a) \end{vmatrix} + \begin{vmatrix} f_1(a) & f_2(a) & f_3(a) \\ g_1(a) & g_2(a) & g_3(a) \\ h_1'(a) & h_2'(a) & h_3'(a) \end{vmatrix}$$

$$F'(a) = D_1 + D_2 + D_3$$

Using eq. (2) we get $D_1 = D_2 = D_3 = 0$ [By the property of determinants that D = 0 if two rows in D are identical]

$\therefore$ F' (a) = 0.

Q. 3. If f(x) = $\log_x$ (ln x), then f '(x) at x = e is ....

Ans. 1/e

Solution. Given that

$$f(x)=\log_x(\ln x)=\frac{\log_e(\log_e x)}{(\log_e x)}$$

$$f'(x)=\frac{\frac{1}{\log_e x}\times\frac{1}{x}\times\log_e x-\frac{1}{x}\log_e(\log_e x)}{(\log_e x)^2}$$

$$=\frac{\frac{1}{x}[1-\log_e(\log_e x)]}{(\log_x x)^2}$$

$$f'(e)=\frac{\frac{1}{e}[1-\log_e(\log_e e)]}{(\log_e e)^2}=\frac{\frac{1}{e}[1-\log_e 1]}{(1)^2}=\frac{1}{e}(1-0)=\frac{1}{e}.$$

Q. 4. The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ **at = 1/2 is ....**

Ans. 4

Solution.

Let $u=\sec^{-1}\left(\frac{1}{2x^2-1}\right);v=\sqrt{1-x^2}$

Then to find $\left.\frac{du}{dv}\right|_{x=1/2}$, we have

$$u=\cos^{-1}(2x^2-1)=2\cos^{-1}x$$

$$\therefore\quad \frac{du}{dx}=\frac{-2}{\sqrt{1-x^2}}\quad\text{and}\quad v=\sqrt{1-x^2}$$

$$\therefore\quad \frac{dv}{dx}=\frac{-x}{\sqrt{1-x^2}}\quad\therefore\quad \frac{du}{dv}=\frac{\frac{-2}{\sqrt{1-x^2}}}{\frac{-x}{\sqrt{1-x^2}}}=\frac{2}{x}$$

$$\therefore\quad \left.\frac{du}{dv}\right|_{x=\frac{1}{2}}=4$$

Q. 5. If f (x) = | x – 2 | and g(x) = f [f(x)], then g'(x) = ... for x > 20

Ans. 1

Solution. f (x) = | x – 2 |

⇒ g (x) = f (f (x)) = | f (x) – 2 | as x > 20

$= || x - 2| -2 | = |x - 2- 2|$ as $x > 20 = | x – 4 |$

$= x – 4$ as $x > 20$

$\therefore g'(x) = 1$

Q. 6. If $xe^{xy} = y + \sin^2 x$, then at x = 0, dy/dx = ...

Ans. 1

Solution. Given : $xe^{xy} = y + \sin^2 x$

Differentiating both sides w. r.to x, we get

$$e^{xy}.1 + xe^{xy}\left(y + x\frac{dy}{dx}\right) = \frac{dy}{dx} + 2\sin x\cos x$$

$$\text{Put } x = 0 \Rightarrow 1 + 0 = \frac{dy}{dx} + 0 \Rightarrow \frac{dy}{dx} = 1$$

True/ False

Q. 1. The derivative of an even function is always an odd function.

Ans. T

Solution. Consider $\phi(x) = \frac{f(x)+f(-x)}{2}$, , which is an even function

$$\text{Now, } \psi(x) = \phi'(x) = \frac{f'(x) - f'(-x)}{2}$$

$$\psi(-x) = \frac{f'(-x) - f'(x)}{2} = -\psi(x) \quad \therefore \psi \text{ is odd.}$$

Subjective Questions of Differentiation

Q.1. Find the derivative of sin (x^2 + 1) with respect to x from first principle.

Ans. 2 x cos (x^2 + 1)

Solution. Let f (x) = sin (x^2 + 1) then

$$f'(x) = \lim_{\delta x \to 0} \frac{\sin[(x+\delta x)^2 + 1] - \sin[x^2+1]}{\delta x}$$

$$\Rightarrow \quad f'(x) = \lim_{\delta x \to 0} 2\cos\left(\frac{(x^2 + (\delta x)^2 + 2x\delta x + 1 + x^2 + 1}{2}\right)$$

$$\frac{\sin\left(\frac{x^2 + (\delta x)^2 + 2x\delta x + 1 - x^2 - 1}{2}\right)}{\delta x}$$

$$= \lim_{\delta x \to 0} \frac{2\cos\left[x^2 + 1 + x\delta x + \frac{(\delta x)^2}{2}\right]\sin\left[\frac{(\delta x)^2 + 2x\,\delta x}{2}\right]}{\delta x\left[\frac{\delta x + 2x}{2}\right]}$$

$$\times \left(\frac{\delta x + 2x}{2}\right)$$

$$= 2\cos(x^2+1) \lim_{\delta x \to 0} \frac{\sin\left[\frac{(\delta x)^2 + 2x\delta x}{2}\right]}{\left[\frac{(\delta x)^2 + 2x\delta x}{2}\right]} \times \left(\frac{\delta x + 2x}{2}\right)$$

$$= 2\cos(x^2+1) \times 1 \times \frac{2x}{2} = 2x\cos(x^2+1)$$

Q.2. Find the derivative of

$$f(x) = \begin{cases} \dfrac{x-1}{2x^2-7x+5} & \text{when } x \neq 1 \\ -\dfrac{1}{3} & \text{when } x = 1 \end{cases}$$

at $x = 1$

Ans. -2/9

Solution.

$$f(x)\begin{cases} \dfrac{x-1}{2x^2-7x+5} & , x \neq 1 \\ -\dfrac{1}{3} & , x = 1 \end{cases}$$

$$\therefore \quad f'(x)|_{x=1} = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0}\left[\frac{\dfrac{1+h-1}{2(1+h)^2-7(1+h)+5}+\dfrac{1}{3}}{h}\right] = \lim_{h\to 0}\frac{\dfrac{h}{2h^2-3h}+\dfrac{1}{3}}{h}$$

$$= \lim_{h\to 0}\frac{\dfrac{1}{2h-3}+\dfrac{1}{3}}{h} = \lim_{h\to 0}\frac{2h}{3h(2h-3)} = \lim_{h\to 0}\frac{2}{3(2h-3)}$$

$$= -2/9$$

Q.3.

Given $y = \dfrac{5x}{3\sqrt{(1-x)^2}} + \cos^2(2x+1)$; Find $\dfrac{dy}{dx}$.

Ans.

$$\frac{5}{3}\cdot\frac{1}{(1-x)^2} - 2\sin(4x+2), \text{ if } x<1 \ ; \ -\frac{5}{3}\cdot\frac{1}{(1-x)^2} - 2\sin(4x+2), \text{ if } x>1$$

Solution. We have, $y = \dfrac{5x}{3|1-x|} + \cos^2(2x+1)$

(Clearly y is not defined at x = 1)

$$\Rightarrow \quad y = \begin{cases} \dfrac{5x}{3(1-x)} + \cos^2(2x+1), & x < 1 \\ \dfrac{5x}{3(x-1)} + \cos^2(2x+1), & x > 1 \end{cases}$$

$$\Rightarrow \quad \frac{dy}{dx} = \begin{cases} \dfrac{5}{3}\left(\dfrac{(1-x)-x(-1)}{(1-x)^2}\right) - 2\sin(4x+2), & x < 1 \\ \dfrac{5}{3}\left(\dfrac{(x-1)-x}{(x-1)^2}\right) - 2\sin(4x+2), & x > 1 \end{cases}$$

$$\text{or} \quad \frac{dy}{dx} = \begin{cases} \dfrac{5}{3}\dfrac{1}{(1-x)^2} - 2\sin(4x+2), & x < 1 \\ -\dfrac{5}{3}\dfrac{1}{(x-1)^2} - 2\sin(4x+2), & x > 1 \end{cases}$$

Q. 4. Let $y = e^{x\sin x^3} + (\tan x)^x$. Find $\dfrac{dy}{dx}$

Ans. $e^{x\sin x^3}\left[\sin x^3 + 3x^3\cos x^3\right] + (\tan x)^x\left[\dfrac{2x}{\sin 2x} + \log\tan x\right]$

Solution. We are given $y = e^{x\sin x^3} + (\tan x)^x$

Here y is the sum of two functions and in the second function base as well as power are functions of x. Therefore we will use logarithmic differentiation here.

Let y = u + v

where $u = e^{x\sin x^3}$ ...(1)

and $v = (\tan x)^x$ ...(2)

$$\therefore \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad ...(3)$$

Differentiating (1) with respect to x, we get

$$\frac{du}{dx} = e^{x\sin x^3} \cdot \frac{d}{dx}(x\sin x^3)$$

$$= e^{x\sin x^3} \cdot [3x^3 \cdot \cos x^3 + \sin x^3]$$

Taking log on both sides on Eqn (2), we get

log v = x log tan x

Differentiating the above with respect to x, we get

$$\frac{1}{v}\frac{dv}{dx} = x \cdot \frac{1}{\tan x} \cdot \sec^2 x + 1 \cdot \log \tan x$$

$$\therefore \quad \frac{dv}{dx} = (\tan x)^x \left[\frac{2x}{\sin 2x} + \log \tan x\right]$$

Substituting the value of $\frac{du}{dx}$ *and* $\frac{dv}{dx}$ in eq^n (3), we get

$$\frac{dy}{dx} = e^{x\sin x^3}[\sin x^3 + 3x^3 \cos x^3] + (\tan x)^x \left[\frac{2x}{\sin 2x} + \log \tan x\right]$$

Q. 5. Let f be a twice differentiable function such that

f ''(x) = – f (x), and f '(x)= g (x), h(x) = [f (x)]^2 + [g (x)]^2

Find h(10) if h(5) = 11

Ans. 11

Solution. Given that f is twice differentiable such that

f '' (x) = – f (x) and f '(x) = g(x)

h (x) = [f (x)]^2 + [g (x)]^2

To find h (10) when h (5) = 11.

Consider h'(x) = 2f f ' + 2gg' = 2f (x) g(x) + 2g(x)f '' (x)

[∴ g(x) = f '(x) ⇒ g' (x) = f ''(x)]

= 2f (x) g (x) + 2g(x) (– f (x))

= 2f (x) g (x) – 2f (x) g (x) = 0

$\therefore \quad h'(x) = 0, \forall x$

⇒ h is a constant function

∴ h (5) = 11 ⇒ h (10) = 11.

Q. 6. If a be a repeated root of a quadratic equation f(x) = 0 and A(x), B(x) and C(x) be polynomials of degree 3, 4 and 5 respectively, then show that $\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$ is **divisible by f(x), where prime denotes the derivatives.**

Solution.

$$\text{Let } F(x) = \begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$$

Given that α is a repeated root of quadratic equation f (x) = 0

∴ We must have f (x) = k (x – α)^2 ; where k is a non- zero real no.

If we put x = α on both sides of eq. (1); we get

$$F(\alpha) = \begin{vmatrix} A(\alpha) & B(\alpha) & C(\alpha) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix} = 0$$

$[\because \; R_1 \text{ and } R_2 \text{ are identical}]$

$\therefore \quad F(\alpha) = 0$

Hence (x – α) is a factor of F(x) Differentiating eq. (1) w.r. to x, we get

$$F'(x) = \begin{vmatrix} A'(x) & B'(x) & C'(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$$

Putting x = α on both sides, we get

$$F'(\alpha) = \begin{vmatrix} A'(\alpha) & B'(\alpha) & C'(\alpha) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix} = 0$$

[as R_1 and R_3 are identical]

⇒ (x – α) is a factor of F'(x) al so. Or we can say $(x - \alpha)^2$ is a factor of F(x).

⇒ F (x) is divisible by f (x).

Q. 7. If x = sec θ – cos θ and y = $\sec^n \theta$ - $\cos^n \theta$, then

show that $(x^2 + 4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2 + 4)$

Solution. We have, x = sec θ – cos θ, y = $\sec^n \theta - \cos^n \theta$

$$\Rightarrow \frac{dx}{d\theta} = \sec\theta\tan\theta + \sin\theta$$

sec θ tan θ + tan θ cos θ = tan θ (sec θ + cos θ)

and $\frac{dy}{d\theta} = n\sec^{n-1}\theta\sec\theta\tan\theta - n\cos^{n-1}\theta(-\sin\theta)$

$= n\sec^n\theta\tan\theta + n\tan\theta\cos^n\theta = n\tan\theta(\sec^n\theta + \cos^n\theta)$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{n\tan\theta(\sec^n\theta + \cos^n\theta)}{\tan\theta(\sec\theta + \cos\theta)}$$

or $\frac{dy}{dx} = \frac{n(\sec^n\theta + \cos^n\theta)}{(\sec\theta + \cos\theta)}$ ...(1)

Also $x^2 + 4 = (\sec\theta - \cos\theta)^2 + 4$

$= \sec^2\theta + \cos^2\theta - 2\sec\theta\cos\theta + 4$

= sec2 θ + cos2 θ + 2

$= (\sec\theta + \cos\theta)^2$ ...(2)

and $y^2 + 4 = \sec n\ \theta - \cos n\ \theta)2 + 4$

$= \sec^{2n}\theta + \cos^{2n}\theta - 2\sec n\ \theta \cos n\ \theta + 4$

$= \sec^{2n}\theta + \cos^{2n}\theta + 2$

$= (\sec n\ \theta + \cos n\ \theta)^2$ ...(3)

Now we have to prove

$$(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$$

$$\text{LHS} = (\sec\theta+\cos\theta)^2 . \frac{n^2(\sec^n\theta+\cos^n\theta)^2}{(\sec\theta+\cos\theta)^2}$$

[Using (1) and (2)]

$= n^2 (\sec^n q + \cos^n q)^2$

$= n^2 (y^2 + 4)$ [From eq. (3)]

= RHS

Q. 8. Find dy/dx at x = – 1, when

$$(\sin y)^{\sin\left(\frac{\pi}{2}x\right)} + \frac{\sqrt{3}}{2}\sec^{-1}(2x) + 2^x \tan(\ln(x+2)) = 0$$

Ans. 0

Solution. We have given the function

$$(\sin y)^{\sin\left(\frac{\pi x}{2}\right)} + \frac{\sqrt{3}}{2}\sec^{-1}(2x) + 2^x\tan[\ln(x+2)] = 0 \quad ...(1)$$

For x = – 1, we have

$$(\sin y)^{\sin\left(-\frac{\pi}{2}\right)} + \frac{\sqrt{3}}{2}\sec^{-1}(-2) + 2^{-1}\tan[\ln(-1+2)] = 0$$

$$\Rightarrow \quad (\sin y)^{-1} + \frac{\sqrt{3}}{2}\left(\frac{2\pi}{3}\right) + \frac{1}{2}\tan 0 = 0 \Rightarrow \frac{1}{\sin y} = -\frac{\pi}{\sqrt{3}}$$

$$\Rightarrow \quad \sin y = -\frac{\sqrt{3}}{\pi}, \text{ when } x = -1 \qquad ....(2)$$

Now Let $u = (\sin y)^{\sin\left(\frac{\pi x}{2}\right)}$

Taking ln on both sides; we get

$$\ln u = \sin\left(\frac{\pi x}{2}\right)\ln\sin y$$

Differentiating both sides with respect to x, we get

$$\frac{1}{u}\frac{du}{dx} = \frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)\ln\sin y + \cot y\frac{dy}{dx}\sin\left(\frac{\pi x}{2}\right)$$

$$\Rightarrow \quad \frac{du}{dx} = (\sin y)^{\sin\left(\frac{\pi x}{2}\right)} \times \left[\frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)\ln\sin y + \sin\left(\frac{\pi x}{2}\right)\cot y\frac{dy}{dx}\right] \text{...(3)}$$

Now differentiating eq. (1), we get

$$\frac{d}{dx}\left[(\sin y)^{\sin\left(\frac{\pi x}{2}\right)}\right] + \frac{\sqrt{3}}{2}\frac{1}{2x\sqrt{4x^2-1}}.2 + 2^x(\ln 2)\tan[(\ln(x+2)] + 2^x\sec^2[\ln(x+2)]\frac{1}{x+2} = 0$$

$$\Rightarrow \quad (\sin y)^{\sin\left(\frac{\pi x}{2}\right)}\left[\frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)\ln\sin y + \sin\left(\frac{\pi x}{2}\right)\cot y\frac{dy}{dx}\right]$$

$$+ \frac{\sqrt{3}}{2x\sqrt{4x^2-1}} + 2^x\ln 2\tan(\ln(x+2)) + \frac{2^x\sec^2[\ln(x+2)]}{x+2} = 0$$

At $x=-1$ and $\sin y=-\frac{\sqrt{3}}{\pi}$, we get

$$\Rightarrow \left(-\frac{\sqrt{3}}{\pi}\right)^{-1}\left[0-(-1)\sqrt{\frac{\pi^2}{3}-1}\left(\frac{dy}{dx}\right)_{x=-1}\right]+\frac{\sqrt{3}}{-2\sqrt{3}}+0+2^{-1}=0$$

$$\Rightarrow -\frac{\pi}{\sqrt{3}\sqrt{3}}\sqrt{\pi^2-3}\left(\frac{dy}{dx}\right)_{x=-1}-\frac{1}{2}+\frac{1}{2}=0 \Rightarrow \left.\frac{dy}{dx}\right]_{x=-1}=0$$

Q. 9. If $y=\frac{ax^2}{(x-a)(x-b)(x-c)}+\frac{bx}{(x-b)(x-c)}+\frac{c}{x-c}+1$,

prove that $\frac{y'}{y}=\frac{1}{x}\left(\frac{a}{a-x}+\frac{b}{b-x}+\frac{c}{c-x}\right)$.

Solution.

$$y=\frac{ax^2}{(x-a)(x-b)(x-c)}+\frac{bx}{(x-b)(x-c)}+\frac{c}{x-c}+1$$

$$=\frac{ax^2}{(x-a)(x-b)(x-c)}+\frac{bx}{(x-b)(x-c)}+\frac{x}{x-c}$$

$$=\frac{ax^2}{(x-a)(x-b)(x-c)}+\left(\frac{b}{x-b}+1\right)\frac{x}{x-c}$$

$$=\frac{ax^2}{(x-a)(x-b)(x-c)}+\frac{x^2}{(x-b)(x-c)}$$

$$=\left(\frac{a}{x-a}+1\right)\frac{x^2}{(x-b)(x-c)}=\frac{x^3}{(x-a)(x-b)(x-c)}$$

$\Rightarrow \log y = 3\log x - \log(x-a) - \log(x-b) - \log(x-c)$

$$\Rightarrow \frac{y'}{y}=\frac{3}{x}-\frac{1}{x-a}-\frac{1}{x-b}-\frac{1}{x-c}$$

$$=\left(\frac{1}{x}-\frac{1}{x-a}\right)+\left(\frac{1}{x}-\frac{1}{x-b}\right)+\left(\frac{1}{x}-\frac{1}{x-c}\right)$$

$$=\frac{a}{x(a-x)}+\frac{b}{x(b-x)}+\frac{c}{x(c-x)}$$

$$=\frac{1}{x}\left[\frac{a}{a-x}+\frac{b}{b-x}+\frac{c}{c-x}\right]$$

Assertion & Reason Type Question

Assertion & Reason Type Question

Q. 1. Let f (x) = 2 + cos x for all real x.

STATEMENT - 1: For each real t, there exists a point c in [t, t + π] such that f '(c) = 0 because

STATEMENT - 2: f (t) = f (t + 2π) for each real t.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True.

Ans. (b)

Solution. Given that f (x) = 2 + cos x which is continuous and differentiable everywhere.

Also f ' (x) = – sin x $\Rightarrow$ f ' (x) = 0 $\Rightarrow$ x = nπ

$\Rightarrow$ There exists c $\in$ [t, t + p] for t $\in$ R

Such that f ' (c) = 0

∴ Statement-1 is true.

Also f (x) being periodic of period 2π, statement-2 is true, but statement-2 is not a

correct explanation of statement-1.

Q. 2. Let f and g be real valued functions defined on interval (–1, 1) such that g" (x) is continuous, g (0) ≠ 0. g'(0) = 0, g ''(0) ≠ 0 , and f (x) = g (x) sin x

STATEMENT - 1: $\lim_{x\to 0}[g(x)\cot x - g(0)\operatorname{cosec} x] = f''(0)$ and

STATEMENT - 2: f '(0) = g(0)
(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1
(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1
(c) Statement - 1 is True, Statement - 2 is False
(d) Statement - 1 is False, Statement - 2 is True

Ans. (a)

Solution. We have f (x) = g(x) sin x

⇒ f '(x) = g'(x) sin x + g(x) cos x

⇒ f '(0) = g'(0) × 0 + g (0) = g(0) [∴ g'(0) = 0]

∴ Statement 2 is correct.

$$\text{Also } f''(0) = \lim_{x\to 0}\frac{f'(x)-f'(0)}{x}$$

$$= \lim_{x\to 0}\frac{g(x)\cos x + g'(x)\sin x - g(0)}{x}$$

$$= \lim_{x\to 0}\frac{g(x)\cos x - g(0)}{x} + \lim_{x\to 0}\frac{g'(x)\sin x}{x}$$

$$= \lim_{x\to 0}\frac{g(x)\cos x - g(0)}{x\times\frac{\sin x}{x}} + \lim_{x\to 0} g'(x)$$

$$= \lim_{x\to 0}\frac{g(x)\cos x - g(0)}{\sin x} + g'(0)$$

$$= \lim_{x\to 0}\left[g(x)\cot(x) - g(0)\operatorname{cosec} x\right] + 0$$

$$= \lim_{x\to 0}\left[g(x)\cot x - g(0)\operatorname{cosec} x\right]$$

∴ Statement 1 is also true and is a correct explanation for statement 2.

Integer Value Correct Type Question

Q. 1. If the function $f(x)=x^3+e^{\frac{x}{2}}$ and $g(x)=f^{-1}(x)$, then the value of g' (1) is

Ans. 2

Solution. Given that f (x) = $x^3+e^{x/2}$ and g (x) = f^{-1} (x) then we should have g of (x) = x

$\Rightarrow$ g (f (x)) = x $\Rightarrow$ $g(x^3+e^{x/2})=x$

Differentiating both sides with respect to x, we get

$$g'(x^3+e^{x/2}).\left(3x^2+e^{x/2}.\frac{1}{2}\right)=1$$

$$\Rightarrow \quad g'(x^3+e^{x/2})=\frac{1}{3x^2+e^{x/2}.\frac{1}{2}}$$

For $x=0$, we get $g'(1)=\frac{1}{1/2}=2$

Q. 2. Let $f(\theta)=\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4}<\theta<\frac{\pi}{4}$.

Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is

Ans. 1

Solution.

$$f(\theta)=\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$$

$$=\sin\left[\sin^{-1}\left(\frac{\sin\theta}{\sqrt{\sin^2\theta+\cos 2\theta}}\right)\right] \quad \left[\because \tan^{-1}\frac{x}{y}=\sin^{-1}\frac{x}{\sqrt{x^2+y^2}}\right]$$

$$=\sin\left[\sin^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)\right]=\tan\theta$$

$$\therefore \frac{df(\theta)}{d\tan\theta}=1.$$